Numbers Game, Minuscule Kac-Moody Representations, Infinite d-Complete Posets

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The Numbers Game on a simple graph was developed to compute the Bruhat order on the elements of a Weyl group. It is a deterministic one player combinatorial game whose moves modify the integer labelings of the fixed graph. It was soon used to classify which Bruhat orders on finite Weyl groups were distributive lattices. That classification is the subject of a plenary talk at the MSJ Tohoku conference that precedes this workshop.

We introduce a notion of "minuscule" for representations of Kac-Moody algebras that generalizes this notion for representations of semisimple Lie algebras. Our ultimate goal for this pair of talks will is to use the Numbers Game to classify these minuscule Kac-Moody representations. No prior knowledge of representation theory will be needed, since our methods are combinatorial.

All of the information concerning a minuscule representation of a semisimple Lie algebra is distilled into its associated colored minuscule poset. The minuscule posets arose in the classification of the Bruhat distributive lattices, since they are the subposets consisting of the "join irreducible" elements in those lattices. Some of the simplest elements of Kac-Moody Weyl groups are the λ-minuscule elements. When the Numbers Game was used to prove that their associated Bruhat orders were also distributive lattices, it was shown that the join irreducibles subposets of those distributive lattices were colored d-complete posets. The d-complete posets are generalizations of minuscule posets which enjoy some of the minuscule properties only in the "upward" direction.

Green took modifications of the axioms for colored d-complete posets as a starting point when he constructed doubly infinite representations of Kac-Moody algebras. He did this in the context of a construction that uses any n-colored poset to create a vector space that is equipped with 2n linear operators. He applied this construction to his doubly infinite colored minuscule posets and showed that the linear operators satisfied the relations required for them to represent the generators of a Kac-Moody algebras. Recently Strayer showed that Green's axioms were necessary conditions for building Kac-Moody representations from doubly infinite colored posets. Strayer introduced two new axioms and used one of them to introduce singly infinite colored d-complete posets. In recent joint work with Strayer, building upon results of Green and McGregor-Dorsey, we use Strayer's viewpoints to classify all finite and infinite minuscule and d-complete posets. After defining minuscule representations of Kac-Moody algebras, with Strayer we use his results and those of Green and McGregor-Dorsey to classify these representations.